Tableau 2. $X/D = 2{,}8$

β	0°	10°	20°	30°	40°	50°	60°	70°
θ_p (°C)	11°	25°	11°7	21°8	10°2	15°	16°1	13°6
$\theta_{0\infty}$ (°C)	22°1	37°1	26°4	36°5	26°3	33°8	37°6	37°7
$\frac{\theta_p\ (°K)}{\theta_{0\infty}\ (°K)}$	0,962 (0,962)	0,961 (0,961)	0,957 (0,957)	0,952 (0,952)	0,946 (0,946)	0,939 (0,939)	0,931 (0,931)	0,922 (0,923)

Tableau 3. $X/D = 3{,}8$

β	0°	10°	20°	30°	40°	50°	60°	70°
θ_p (°C)	27°1	25°5	17°4	16°4	15°3	17°2	19°1	15°3
$\theta_{0\infty}$ (°C)	38°4	37°	29°7	30°	30°9	35°1	39°8	38°
$\frac{\theta_p\ (°K)}{\theta_{0\infty}\ (°K)}$	0,964 (0,963)	0,962 (0,961)	0,959 (0,958)	0,955 (0,953)	0,949 (0,947)	0,942 (0,940)	0,934 (0,932)	0,927 (0,924)

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HEAT TRANSFER WITH UNSYMMETRICAL THERMAL BOUNDARY CONDITIONS

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Recently, Hatton and Quarmby [1] presented the results of a comprehensive analytical study of the effects of axially varying and unsymmetrical boundary conditions on heat transfer in turbulent flow between parallel plates. These authors made a thorough investigation of heat transfer in the entrance and fully developed regions covering a wide range of Reynolds number, Prandtl number and thermal situations. The flow was assumed to be fully developed in the hydrodynamic sense.

In their solution of the simplified energy equation for the temperature, Hatton and Quarmby used the separation-of-variables technique† which results in an eigen-

† This method is possible if the velocity (and consequently the eddy diffusivity) is a function of wall distance only. It is therefore restricted to the case of a flow which is developed hydrodynamically.

value problem. They employed the velocity distribution of Deissler [2], but, realizing that in the unsymmetrical heat-transfer case there would be significant temperature gradients in the central region, they modified the eddy diffusivity (ϵ) profile. They assumed a constant value of ϵ over the middle half of the channel on the grounds that this was in closer agreement with experimental observations. For convenience, their results were shown diagrammatically as local Nusselt number versus axial position.

In the present note, a comparison is made between Hatton and Quarmby's analysis and one which has been made by the present writers. To some extent, the comparison serves to check the results of both forms of investigation and brings to light the effect of the important assumption concerning eddy diffusivity in asymmetrical heat transfer. While the basic energy equation and the forms of the velocity distributions of the two analyses are essentially the same, the techniques of calculating the Nusselt numbers are different. We have employed the boundary-layer model making a calculation for the temperature field and hence Nusselt number for various thicknesses of the thermal boundary layer.

In a previous paper [3], in which the present authors were concerned with the thermal entry length for uniform heat flux at one wall (the other wall being adiabatic) expressions for the temperature are presented. These expressions were derived assuming linear heat flux across the thermal boundary layer and an eddy diffusivity distribution as given by Deissler [2]. It is a easy step to calculate the local Nusselt number, Nu, which is given by

$$Nu = \frac{-2s^+\left[\dfrac{d}{dy^+}(T_\delta^+ - T^+)\right]_{y^+=0}}{\int_0^{s^+} u^+ T^+ \, dy^+ \Big/ \int_0^{s^+} u^+ \, dy^+} \quad (1)$$

where s is the channel width, δ is the thermal boundary lay thickness and u^+, y^+, have their usual meanings. T^+ is $[(T_w - T)c\,\tau_w/qu_\tau]$, where T_w is the local wall temperature. The corresponding Reynolds number of the flow is

$$Re = 4\int_0^{s^+/2} u^+ \, dy^+. \quad (2)$$

It will be observed that the expressions for the Nusselt number according to Hatton and Quarmby (their equation 18) and equation (1) are different in form. Differences in the numerical result must also be expected on account of the different eddy diffusivity distributions which have been used. In the present study, we have used an eddy diffusivity which is a linear function of $y^+ (y^+ > 26)$. This is compatible with constant shear-stress distribution. Hatton and Quarmby use the more realistic two-part distribution [their equation (3), $y^+ > 26$] in conjunction with a linear variation of shear stress. The latter authors therefore assume a smaller eddy diffusivity, and their equation will, in general, predict a smaller heat-transfer coefficient than that given by equation (1). The difference in the two results will also be a function of the Reynolds and Prandtl numbers. In the case of fluids of large Prandtl number, the difference will be least marked because the temperature changes in the middle of the channel are smaller with such fluids. The effect of Reynolds number is more difficult to assess.

The features of the two analyses as described previously are illustrated in Fig. 1, where the Nusselt number is plotted against axial position. Hatton and Quarmby's data for the Prandtl number of 0·72 can be obtained by graphical interpolations using their plots for $Pr = 0{\cdot}1$, 1 and 10. Nusselt number for these three values of the Prandtl number were also obtained by a private communication with the same authors. The comparison is restricted to values of $Pr \doteq 1$ because of the use of a simplified expression for ϵ $(y^+ < 26)$ in the boundary layer method.

The Reynolds number given in Fig. 1 are taken from Hatton and Quarmby's paper where their $Re - s^+/2$ relationship is tabulated. Having chosen a value of s^+ which is suitable for computational purposes, the value of Re is determined from equation (2). While we use the same three values of the Reynolds number for the purpose of comparison, it should be realized that our s^+ values are not quite the same as those of Hatton and Quarmby because a different $u^+(y^+)$ relation has been used.

It is to be observed that over the whole range of Reynolds numbers, the "fully developed" Nusselt number calculated with our method is larger. (The largest difference occurs at the largest Reynolds number, Re = 49,4576.) In the developed region, the smaller heat-transfer coefficient of Hatton and Quarmby is compatible with their smaller eddy-diffusivity values. In the entry region, the greatest divergence between the two predictions occurs at the smallest Reynolds number, i.e. at $Re = 7096$. There is, however, excellent agreement at very small values of X^+ for all Reynolds numbers and this is to be expected for the following important reasons In the predominantly viscous region near the wall, our expression for ϵ [viz. $(0{\cdot}109)^2 uy$] is a very good approximation to Hatton and Quarmby's value [their equation (3)] for gas flows. Therefore, where the thermal boundary layer is confined to the region $y^+ < 26$ (correspondingly small X^+), both analyses should predict practically the same heat-transfer coefficient. Coincidence of the curves near $X^+ = 1$ for all Reynolds number is evidence of this. Further evidence of the correctness of the geometry of the curves of both analyses is to be found in a study of their slopes. It will be remembered that Hatton and Quarmby use a constant value of eddy diffusivity over the middle half of the duct. When the thermal boundary layer has grown to $(s/4)$, further growth to the "developed" condition will be less rapid according to their analysis. An examination of the curvature of the curves in Fig. 1 shows the effect of this on the distribution of the local heat-transfer coefficient. The Hatton and Quarmby analysis predicts a smaller heat-transfer coefficient which is attained in a longer entry length.

For completeness, the familiar recommendation $Nu = 0{\cdot}023 \cdot Re^{0{\cdot}8} Pr^{0{\cdot}4}$ for the fully developed heat-transfer

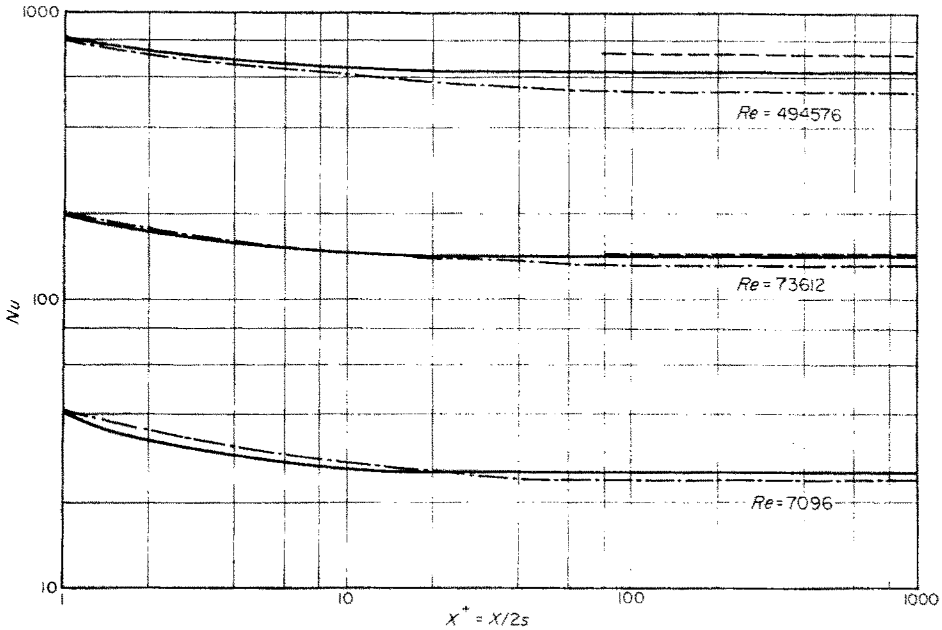

FIG. 1. Nu vs. X^+, one side insulated, q = const., $Pr = 0{\cdot}72$.

———— Present analysis —·—·— Hatton *et al.*

— — — $Nu = 0{\cdot}023 Re^{0{\cdot}8} Pr^{0{\cdot}4}$ ($Re > 10000$)

coefficient has been included. This equation applies to symmetrical heat transfer in pipe flow, and is sometimes used for other geometries on the understanding that the hydraulic mean diameter is applicable.

The greater the eddy diffusion in the heat-transfer process, the shorter will be the entrance length. It is difficult to define the entrance length quantitatively, and the determination of the position of $Nu \rightarrow Nu_\infty$ by the two analyses can only be made approximately. For the present thermal boundary conditions, Hatton and Quarmby's value for the thermal entry length can be estimated from Fig. 1, 2 and 3 of their paper. The value of the thermal entry length according to our boundary-layer model is less for the reasons given previously. For the practical purposes, the difference in the two results is insignificant.

The two analyses require extensive numerical computation, and in view of this, the agreement between the results suggests reliable evaluation of the problem in both studies. Those numerical differences which do exist are readily accounted for by careful consideration of the assumptions used in the analyses.

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